

Discutir y resolver los siguientes sistemas:

$$\textcircled{1} \begin{cases} x+2y+3z=3 \\ 4x+ay+z=4 \\ -6x-6y+4z=-2 \end{cases}$$

$$\textcircled{2} \begin{cases} mx+y=2-2m \\ x+my=m-1 \end{cases}$$

$$\textcircled{3} \begin{cases} 10x+5y=7 \\ 8x-10y=0 \\ ax+(a-1)y=5 \end{cases}$$

$$\textcircled{4} \begin{cases} 2x+y-z=a-4 \\ (a-6)y+3z=0 \\ (a+1)x+2y=3 \end{cases}$$

$$\textcircled{5} \begin{cases} x+2y=5 \\ 3x+ay=a \\ 5x+ay=7 \end{cases}$$

$$\textcircled{6} \begin{cases} x+y+z=3 \\ x-y=0 \\ x+2y+z=4 \\ mx+3my+z=m \end{cases}$$

$$\textcircled{7} \begin{cases} 4x+12y+4z=0 \\ 2x-13y+2z=0 \\ (a+2)x-12y+12z=0 \end{cases}$$

$$\textcircled{8} \begin{cases} x+y=1 \\ ay+z=0 \\ x+(a+1)y+az=a+1 \end{cases}$$

$$\textcircled{9} \begin{cases} 4x-4z=0 \\ x-y+az=0 \\ -x-ay-z=0 \end{cases}$$

$$\textcircled{10} \begin{cases} 2x+y+z=3 \\ y-z=-1 \\ 2x-y+az=b \end{cases}$$

$$\textcircled{11} \begin{cases} 6x+4y+2az=2 \\ ax+y-z=2 \\ 5x+3y+3z=2a \end{cases}$$

$$\textcircled{12} \begin{cases} x-y+z=a \\ 3x+4y-5z=6 \\ x-y+bz=2 \end{cases}$$

$\textcircled{13}$ Discute el sistema: $\begin{cases} x+3y-2z=-3 \\ 3x-y+z=2 \\ ax+2y-z=-1 \end{cases}$ Resuelve para $a=4$.

$\textcircled{14}$ Discute el sistema: $\begin{cases} ax-y+2z=1 \\ x-2y=0 \\ ax+y-z=1 \end{cases}$ Resuelve para $a=1$

$\textcircled{15}$ Determina, si existe, los valores de a, b para el que el sistema: $\begin{cases} 3x-y+2z=1 \\ x+4y+z=b \\ 2x-5y+az=-2 \end{cases}$ es compatible indeterminado. Resuelve para dicho valor de a .

$\textcircled{16}$ Discute el sistema $\begin{cases} x+y+2z=2 \\ 2x-y+3z=2 \\ 5x-y+az=6 \end{cases}$ y resuelve para $a=0$

Ej 1
$$\begin{cases} x+2y+3z=3 \\ 4x+ay+z=4 \\ -6x-6y+4z=-2 \end{cases} \quad |A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & a & 1 \\ -6 & -6 & 4 \end{vmatrix} = 4a - 72 - 12 + 18a + 6 - 32 = 22a - 110 = 0$$

$$a = \frac{110}{22} = 5$$

Si $a \neq 5$ $\Rightarrow |A| \neq 0 \Rightarrow$ $\left. \begin{array}{l} \text{Podemos resolver por Cramer} \\ \text{rang } A = \text{rang } A^* = 3 \end{array} \right\}$ S. Comp Deter

$$x = \frac{\begin{vmatrix} 3 & 2 & 3 \\ 4 & a & 1 \\ -2 & -6 & 4 \end{vmatrix}}{22a - 110} = \frac{12a - 72 - 4 + 6a - 32 + 18}{22a - 110} = \frac{18a - 90}{22(a-5)} = \frac{18(a-5)}{22(a-5)} = \frac{9}{11}$$

$$y = \frac{\begin{vmatrix} 1 & 3 & 3 \\ 4 & 4 & 1 \\ -6 & -2 & 4 \end{vmatrix}}{22a - 110} = \frac{16 - 24 - 18 + 72 + 2 - 48}{22a - 110} = \frac{0}{22a - 110} = 0$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 3 \\ 4 & a & 4 \\ -6 & -6 & -2 \end{vmatrix}}{22a - 110} = \frac{-2a + 72 - 48 + 18a + 24 + 16}{22a - 110} = \frac{16a - 80}{22a - 110} = \frac{16(a-5)}{22(a-5)} = \frac{8}{11}$$

S. Compatible Determinado Sol $(\frac{9}{11}, 0, \frac{8}{11})$

Si $a = 5$ el sistema es: $\begin{cases} x+2y+3z=3 \\ 4x+5y+z=4 \\ -6x-6y+4z=-2 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 4 & 5 & 1 & 4 \\ -6 & -6 & 4 & -2 \end{array} \right) \begin{array}{l} \xrightarrow{F_2 - 4F_1} \\ \xrightarrow{F_3 + 6F_1} \end{array}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -3 & -11 & -8 \\ 0 & 6 & 22 & 16 \end{array} \right) \xrightarrow{F_3 + 2F_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -3 & -11 & -8 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{array}{l} x+2y+3z=3 \\ -3y-11z=-8 \end{array} \quad y = \frac{-8+11z}{-3} = \frac{8-11z}{3}$$

$$x + \frac{8-11z}{3} + 3z = 3 \Rightarrow x = 3 - \frac{8-11z}{3} - 3z = \frac{9-8+11z-9z}{3} = \frac{-7+13z}{3}$$

S. Compatible Indeterminado Sol $(\frac{-7+13z}{3}, \frac{8-11z}{3}, z)$

Ej 2
$$\begin{cases} mx+y=2-2m \\ x+my=m-1 \end{cases} \quad |A| = \begin{vmatrix} m & 1 \\ 1 & m \end{vmatrix} = m^2 - 1 = 0 \quad m = \pm 1$$

Si $m \neq 1$ y $m \neq -1 \Rightarrow$ $\left. \begin{array}{l} |A| \neq 0 \\ \text{rang } A = \text{rang } A^* \end{array} \right\}$ S. Comp. Determ. Ruffini

$$x = \frac{\begin{vmatrix} 2-2m & 1 \\ m-1 & m \end{vmatrix}}{m^2-1} = \frac{2m-2m^2-m+1}{m^2-1} = \frac{-3m^2+2m+1}{m^2-1} \stackrel{\text{Ruffini}}{=} \frac{(3m-1)(m-1)}{(m-1)(m+1)} = \frac{-3m-1}{m+1}$$

$$y = \frac{\begin{vmatrix} m & 2-2m \\ 1 & m-1 \end{vmatrix}}{m^2-1} = \frac{m^2-m-2+2m}{m^2-1} = \frac{m^2+m-2}{m^2-1} \stackrel{\text{Ruffini}}{=} \frac{(m-1)(m+2)}{(m-1)(m+1)} = \frac{m+2}{m+1}$$

$$m \neq 1 \text{ y } m \neq -1 \Rightarrow \text{S. Comp. Determinado Sol: } \left(\frac{-3m-1}{m+1}, \frac{m+2}{m+1} \right) \quad (5)$$

$$\boxed{\text{Si } m=1} \quad \begin{cases} x+y=0 \\ x+y=0 \end{cases} \left. \begin{array}{l} y=-x \\ \end{array} \right\} \text{Sol } (x, -x) \text{ S. Comp. Indeterminado}$$

$$\boxed{\text{Si } m=-1} \quad \begin{cases} -x+y=4 \\ x-y=-2 \end{cases} \left. \begin{array}{l} x-y=-4 \\ x-y=-2 \end{array} \right\} \text{Contradicción. S. Incompatible}$$

$$\boxed{\text{Ej 3}} \quad \begin{cases} 10x+5y=7 \\ 8x-10y=0 \\ ax+(a-1)y=5 \end{cases} \left. \begin{array}{l} |A^*| = \begin{vmatrix} 10 & 5 & 7 \\ 8 & -10 & 0 \\ a & a-1 & 5 \end{vmatrix} = -500 + 56a - 56 + 70a - 200 \\ = -756 + 126a = 0; a = \frac{756}{126} = 6 \end{array} \right\}$$

$$\boxed{\text{Si } a \neq 6} \Rightarrow |A^*| \neq 0 \Rightarrow \text{rang } A^* = 3 \Rightarrow \text{S. Incompatible}$$

$$\boxed{\text{Si } a=6} \quad \begin{cases} 10x+5y=7 \\ 8x-10y=0 \\ 6x+5y=5 \end{cases} \left. \begin{array}{l} +10x+5y=7 \\ -6x-8y=-5 \\ \hline 4x=2 \Rightarrow x=\frac{2}{4}=\frac{1}{2} \end{array} \right\} \begin{array}{l} +6 \cdot \frac{1}{2} + 5y = 5 \\ 5y = 2 \\ y = \frac{2}{5} \end{array}$$

$$\text{Comprobamos en } E_2: 8 \cdot \frac{1}{2} - 10 \cdot \frac{2}{5} = 4 - 4 = 0 \text{ Verdadera}$$

$$\text{Si } a=6 \text{ Sol } \left(\frac{1}{2}, \frac{2}{5} \right) \text{ S. Compatible Determinado}$$

$$\boxed{\text{Ej 4}} \quad \begin{cases} 2x+y-z=a-4 \\ (a-6)y+3z=0 \\ (a+1)x+2y=3 \end{cases} \left. \begin{array}{l} |A| = \begin{vmatrix} 2 & 1 & -1 \\ 0 & a-6 & 3 \\ a+1 & 2 & 0 \end{vmatrix} = 3(a+1) + (a+1)(a-6) - 12 = \\ = 3a+3 + a^2-6a+a-6-12 = a^2-2a-15=0 \end{array} \right\}$$

$$a = \frac{2 \pm \sqrt{4+60}}{2} = \frac{2 \pm 8}{2} < \begin{matrix} 5 \\ -3 \end{matrix}$$

$$\boxed{\text{Si } a \neq -3 \text{ y } a \neq 5} \Rightarrow |A| \neq 0 \Rightarrow \text{S. Compatible determinado}$$

$$x = \frac{\begin{vmatrix} a-4 & 1 & -1 \\ 0 & a-6 & 3 \\ 3 & 2 & 0 \end{vmatrix}}{a^2-2a-15} = \frac{9+3(a-6)-6(a-4)}{a^2-2a-15} = \frac{-3a+15}{a^2-2a-15} = \frac{-3(a-5)}{(a-5)(a+3)} = \frac{-3}{a+3}$$

$$y = \frac{\begin{vmatrix} 2 & a-4 & -1 \\ 0 & 0 & 3 \\ a+1 & 3 & 0 \end{vmatrix}}{a^2-2a-15} = \frac{3(a-4)(a-1)-18}{a^2-2a-15} = \frac{3a^2-9a-30}{a^2-2a-15} = \frac{3(a-5)(a+2)}{(a-5)(a+3)} = \frac{3a+6}{a+3}$$

$$z = \frac{\begin{vmatrix} 2 & 1 & a-4 \\ 0 & a-6 & 0 \\ a+1 & 2 & 3 \end{vmatrix}}{a^2-2a-15} = \frac{6(a-6)-(a+1)(a-6)(a-4)}{a^2-2a-15} = \frac{(a-6)[6-(a+1)(a-4)]}{(a-5)(a+3)}$$

$$= \frac{(a-6)(-a^2+3a+10)}{(a-5)(a+3)} = \frac{(a-6)(a-5)(-a-2)}{(a-5)(a+3)} = \frac{-(a-6)(a+2)}{a+3}$$

$$\text{Sol } \left(\frac{-3}{a+3}, \frac{3a+6}{a+3}, \frac{-(a-6)(a+2)}{a+3} \right)$$

Si $a = -3$
$$\left. \begin{array}{l} 2x+y-z = -7 \\ -9y+3z = 0 \\ -2x+2y = 3 \end{array} \right\} \left(\begin{array}{ccc|c} 2 & 1 & -1 & -7 \\ 0 & -9 & 3 & 0 \\ -2 & 2 & 0 & 3 \end{array} \right) \xrightarrow{F_2+F_3, F_1+F_3} \left(\begin{array}{ccc|c} 2 & 1 & -1 & -7 \\ 0 & -9 & 3 & 0 \\ 0 & 3 & -1 & -4 \end{array} \right) \xrightarrow{F_2+3F_3} \textcircled{6}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & -7 \\ 0 & 0 & 0 & 12 \\ 0 & 3 & -1 & -4 \end{array} \right) E_2 : 0 = 12 \text{ Contradicción} \Rightarrow \text{S. Incompatible}$$

Si $a = 5$
$$\left. \begin{array}{l} 2x+y-z = 1 \\ -y+3z = 0 \\ 6x+2y = 3 \end{array} \right\} \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -1 & 3 & 0 \\ 6 & 2 & 0 & 3 \end{array} \right) \xrightarrow{F_3-3F_1} \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -1 & 3 & 0 \\ 0 & -1 & 3 & 0 \end{array} \right) \xrightarrow{F_3-F_2} \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\left. \begin{array}{l} 2x+y-z = 1 \\ -y+3z = 0 \end{array} \right\} \begin{cases} x = \frac{1-y+z}{2} = \frac{1-3z+z}{2} = \frac{1-2z}{2} \\ y = 3z \end{cases} \text{Sol} \left(\frac{1-2z}{2}, 3z, z \right)$$

S. Compatible Indeterminado

Ej 5

$$\left. \begin{array}{l} x+2y = 5 \\ 3x-ay = a \\ 5x+ay = 7 \end{array} \right\} |A^*| = \begin{vmatrix} 1 & 2 & 5 \\ 3 & -a & a \\ 5 & a & 7 \end{vmatrix} = -7a + 15a + 10a + 25a - a^2 - 42 = -a^2 + 43a - 42 = 0$$

$$a = \frac{-43 \pm \sqrt{43^2 - 4 \cdot 42}}{-2} = \frac{-43 \pm 41}{-2} \begin{cases} \frac{-2}{-2} = 1 \\ \frac{-84}{-2} = 42 \end{cases}$$

Si $a \neq 1$ y $a \neq 42$ $\Rightarrow |A^*| \neq 0 \Rightarrow \text{rang } A^* = 3 \Rightarrow \text{S. Incompatible}$

Si $a = 1$
$$\left. \begin{array}{l} x+2y = 5 \\ 3x-y = 1 \\ 5x+y = 7 \end{array} \right\} \begin{array}{l} 3x-y = 1 \\ 3-y = 1 \Rightarrow y = 2 \\ \underline{5x+y = 7} \\ 8x = 8 \Rightarrow x = 1 \end{array}$$

Sistema compatible determinado Sol (1, 2)

Si $a = 42$
$$\left. \begin{array}{l} x+2y = 5 \\ 3x-42y = 42 \\ 5x+42y = 7 \end{array} \right\} \begin{array}{l} 3x-42y = 42 \\ \underline{5x+42y = 7} \\ 8x = 49 \\ x = \frac{49}{8} \end{array}$$

$$\begin{array}{l} 5 \cdot \frac{49}{8} + 42y = 7 \\ 245 + 336y = 56 \\ 336y = -189 \\ y = \frac{-189}{336} = \frac{-9}{16} \end{array}$$

Comprobamos en E_1 : $\frac{49}{8} - \frac{18}{16} = \frac{98-18}{16} = \frac{80}{16} = 5$ Verdadera

S. Compatible determinado Sol $\left(\frac{49}{8}, \frac{-9}{16} \right)$

Ej 6
$$\begin{cases} x+y+z=3 \\ x-y=0 \\ x+2y+z=4 \\ mx+3my+z=m \end{cases}$$
 Resolvemos
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & 0 & 0 \\ 1 & 2 & 1 & 4 \end{array} \right) \begin{array}{l} F_2 - F_1 \\ F_3 - F_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -1 & -3 \\ 0 & 1 & 0 & 1 \end{array} \right) \begin{array}{l} y=1 \\ 2y+z=3 \\ z=1 \\ x=1 \end{array}$$

$mx+3my+z=m$

Comprobamos en E_4 : $m+3m+1=m \Rightarrow 3m=1 \Rightarrow m=1/3$

Si $m=1/3$ se cumplirán al tiempo las 4 igualdades para $x=1, y=1, z=1 \Rightarrow$ S. Compatible determinado Sol $(1, 1, 1)$

Si $m \neq 1/3$ E_4 será falsa \Rightarrow S. Incompatible

Ej 7
$$\begin{cases} 4x+12y+4z=0 \\ 2x-13y+2z=0 \\ (a+2)x-12y+12z=0 \end{cases}$$
 Es un sistema homogéneo, luego va a ser siempre compatible. Lo simplificamos

$$\begin{cases} x+3y+z=0 \\ 2x-13y+2z=0 \\ (a+2)x-12y+12z=0 \end{cases}$$

$$|A| = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -13 & 2 \\ a+2 & -12 & 12 \end{vmatrix} = -156 - 24 + 6(a+2) + 13(a+2) + 24 - 72$$

$$= -156 - 24 + 6a + 12 + 13a + 26 + 24 - 72 = 19a - 190 = 0$$

$$a = \frac{190}{19} = 10.$$

Si $a \neq 10 \Rightarrow |A| \neq 0 \Rightarrow$ S. Compatible determinado. Sol. trivial $(0, 0, 0)$

8

$$\boxed{\text{Si } a=10} \quad \left. \begin{array}{l} x+3y+z=0 \\ 2x-13y+2z=0 \\ x-y+z=0 \end{array} \right\} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 2 & -13 & 2 & 0 \end{array} \right) \xrightarrow{F_2-F_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & -11 & 0 & 0 \end{array} \right) \xrightarrow{F_3-2F_1}$$

$$\begin{aligned} x-y+z &= 0 & x &= -z \\ 4y &= 0 & \Rightarrow y &= 0 \\ -11y &= 0 & \Rightarrow y &= 0 \end{aligned}$$

Sol $(-z, 0, z) \Rightarrow$ S. Compatible Indeterminado

Ej 8 $\left. \begin{array}{l} x+y=1 \\ ay+z=0 \\ x+(a+1)y+az=a+1 \end{array} \right\} \begin{array}{l} |A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & a & 1 \\ 1 & a+1 & a \end{vmatrix} = a^2+1-(a+1) = a^2-a = 0 \\ \begin{array}{l} a=0 \\ a=1 \end{array} \end{array}$

$\boxed{\text{Si } a \neq 0 \text{ y } a \neq 1} \Rightarrow |A| \neq 0 \Rightarrow$ S. Compatible determinado

$\boxed{\text{Si } a=0} \quad \left. \begin{array}{l} x+y=1 \\ z=0 \\ x+y=1 \end{array} \right\} \begin{array}{l} y=1-x \\ z=0 \end{array} \quad \text{Sol } (x, 1-x, 0) \Rightarrow$ S. Comp. Indeterminado

$\boxed{\text{Si } a=1} \quad \left. \begin{array}{l} x+y=1 \\ y+z=0 \\ x+2y+z=2 \end{array} \right\} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 \end{array} \right) \xrightarrow{F_3-F_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)$

$E_3: 0=1 \Rightarrow$ Contradicción \Rightarrow S. Incompatible

Solución del caso $a \neq 0$ y $a \neq 1$ (No la piden)

$$x = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 0 & a & 1 \\ a+1 & a+1 & a \end{vmatrix}}{a^2-a} = \frac{a^2+a+1-a-1}{a^2-a} = \frac{a^2}{a(a-1)} = \frac{a}{a-1}$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 1 & a+1 & a \\ 1 & a+1 & a+1 \end{vmatrix}}{a^2-a} = \frac{1-a-1}{a^2-a} = \frac{-a}{a(a-1)} = \frac{-1}{a-1}$$

Sol $(\frac{a}{a-1}, \frac{-1}{a-1}, \frac{a}{a-1})$

$$z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 1 & a+1 & a+1 \end{vmatrix}}{a^2-a} = \frac{a^2+a-a}{a^2-a} = \frac{a^2}{a(a-1)} = \frac{a}{a-1}$$

Ej 10 $\left. \begin{array}{l} 2x+y+z=3 \\ y-z=-1 \\ 2x-y+az=b \end{array} \right\} |A| = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & -1 & a \end{vmatrix} = 2a-2-2-2 = 2a-6 = 0 \Rightarrow a=3$

$\boxed{\text{Si } a \neq 3} \Rightarrow |A| \neq 0 \Rightarrow$ S. Comp. Determinado ($\forall b$)

Solución: (No la piden)

$$x = \frac{\begin{vmatrix} 3 & 1 & 1 \\ -1 & 1 & -1 \\ b-1 & a & a \end{vmatrix}}{2a-6} = \frac{3a+1-b-b-3+a}{2(a-3)} = \frac{4a-2b-2}{2(a-3)} = \frac{2a-b-1}{a-3}$$

(9)

$$y = \frac{\begin{vmatrix} 2 & 3 & 1 \\ 0 & -1 & -1 \\ 2 & b & a \end{vmatrix}}{2a-6} = \frac{-2a-6+2+2b}{2a-6} = \frac{-2a+2b-4}{2(a-3)} = \frac{-a+b-2}{a-3}$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 3 \\ 0 & 1 & -1 \\ 2 & -1 & b \end{vmatrix}}{2a-6} = \frac{2b-2-6-2}{2a-6} = \frac{2b-10}{2a-6} = \frac{b-5}{a-3}$$

$$\text{Sol} \left(\frac{2a-b-1}{a-3}, -\frac{a+b-2}{a-3}, \frac{b-5}{a-3} \right)$$

Si $a=3$

$$\begin{cases} 2x+y+z=3 \\ y-z=-1 \\ 2x-y+3z=b \end{cases} \rightarrow \left(\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 2 & -1 & 3 & b \end{array} \right) \xrightarrow{F_2-F_1} \left(\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & 2 & b-3 \end{array} \right) \xrightarrow{F_3+2F_2} \left(\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & b-5 \end{array} \right)$$

Si $a=3$ y $b \neq 5$ $E_3: 0 = b-5$ Contradicción si $b \neq 5$
S. Incompatible

Si $a=3$ y $b=5$

$$\begin{cases} 2x+y+z=3 \\ y-z=-1 \end{cases} \rightarrow \begin{cases} 2x+z-1+z=3 \Rightarrow 2x=4-2z \\ y=2-1 \\ x=2-z \end{cases}$$

Sol $(2-z, 2-1, z)$ S. Compatible indeterminado

Ej 13

$$\begin{cases} x+3y-2z=-3 \\ 3x-y+z=2 \\ ax+2y-z=-1 \end{cases} \rightarrow |A| = \begin{vmatrix} 1 & 3 & -2 \\ 3 & -1 & 1 \\ a & 2 & -1 \end{vmatrix} = 1-12+3a-2a-2+9 = a-4.$$

Ej 14 **Si $a \neq 4$** $\Rightarrow |A| \neq 0 \Rightarrow$ S. Compatible determinado

Si $a=4$

$$\begin{cases} x+3y-2z=-3 \\ 3x-y+z=2 \\ 4x+2y-z=-1 \end{cases} \rightarrow \left(\begin{array}{ccc|c} 1 & 3 & -2 & -3 \\ 3 & -1 & 1 & 2 \\ 4 & 2 & -1 & -1 \end{array} \right) \xrightarrow{F_2-3F_1} \left(\begin{array}{ccc|c} 1 & 3 & -2 & -3 \\ 0 & -10 & 7 & 11 \\ 0 & -10 & 7 & 11 \end{array} \right) \xrightarrow{F_3-F_2} \left(\begin{array}{ccc|c} 1 & 3 & -2 & -3 \\ 0 & -10 & 7 & 11 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{cases} -10y+7z=11 \\ x+3y-2z=-3 \end{cases} \rightarrow \begin{cases} y = \frac{7z-11}{10} \\ x = -3-3y+2z = -3 - \frac{21z-33}{10} + 2z = \frac{-30-21z+33+20z}{10} = \frac{3-z}{10} \end{cases}$$

Sol $\left(\frac{3-z}{10}, \frac{7z-11}{10}, z \right)$ S. Compatible indeterminado.

Ej 16

$$\begin{cases} x+y+2z=2 \\ 2x-y+3z=2 \\ 5x-y+az=6 \end{cases} \rightarrow |A| = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & a \end{vmatrix} = -a-4+15+10+3-2a = -3a+24=0$$

$a=8$

Si $a \neq 8$ $\Rightarrow |A| \neq 0 \Rightarrow$ S. Compatible determinado

(10)

Si $a=8$ $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & -1 & 3 & 2 \\ 5 & -1 & 8 & 6 \end{array} \right) \xrightarrow{\substack{F_2-2F_1 \\ F_3-5F_1}} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & -6 & -4 & -4 \end{array} \right) \xrightarrow{F_3-2F_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right)$

$-3y - z = -2 \Rightarrow z = 2 - 3y$
 $x + y + 2z = 2 \Rightarrow x = 2 - y - 2z = 2 - y - 2(2 - 3y) = -2 + 5y$

Para $a=0$ $\left. \begin{array}{l} x + y + 2z = 2 \\ 2x - y + 3z = 2 \\ 5x - y = 6 \end{array} \right\} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & -1 & 3 & 2 \\ 5 & -1 & 0 & 6 \end{array} \right) \xrightarrow{\substack{F_2-2F_1 \\ F_3-5F_1}} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & -6 & -10 & -4 \end{array} \right) \xrightarrow{F_3-2F_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & -8 & 0 \end{array} \right)$

$x + y + 2z = 2$ $x + \frac{2}{3} = 2 \Rightarrow x = 2 - \frac{2}{3} = \frac{4}{3}$
 $-3y - z = -2$ $-3y - 2 = -2 \Rightarrow y = \frac{2}{3}$
 $-8z = 0 \Rightarrow z = 0$ Sol $(\frac{4}{3}, \frac{2}{3}, 0)$ S. Comp. Determin.

Ej 9

$\left. \begin{array}{l} 4x - 4z = 0 \\ x - y + az = 0 \\ -x - ay - z = 0 \end{array} \right\} \begin{array}{l} x - z = 0 \\ x - y + az = 0 \\ -x - ay - z = 0 \end{array}$ Sistema homogéneo.
 Sistema compatible $\forall a$.

$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 1 & -1 & a \\ -1 & -a & -1 \end{vmatrix} = 1 + a + 1 + a^2 = a^2 + a + 2 = 0$
 $a = \frac{-1 \pm \sqrt{1-8}}{2}$ no es real $\Rightarrow a^2 + a + 2 \neq 0 \forall a$.

Luego $\forall a$ $|A| \neq 0 \Rightarrow$ S. compatible determinado.
 y solo puede la solución trivial $(0, 0, 0)$

Ej 12

$\left. \begin{array}{l} ax - y + 2z = 1 \\ x - 2y = 0 \\ ax + y - z = 1 \end{array} \right\} |A| = \begin{vmatrix} a & -1 & 2 \\ 1 & -2 & 0 \\ a & 1 & -1 \end{vmatrix} = 2a + 2 + 4a - 1 = 6a + 1 \Rightarrow a = -1/6$

Si $a \neq -1/6 \Rightarrow |A| \neq 0 \Rightarrow$ S. Compatible Determinado

Si $a = -1/6$ $\left. \begin{array}{l} -1/6 x - y + 2z = 1 \\ x - 2y = 0 \\ -1/6 x + y - z = 1 \end{array} \right\} \left(\begin{array}{ccc|c} -1 & -6 & 12 & 6 \\ 1 & -2 & 0 & 0 \\ -1 & 6 & -6 & 6 \end{array} \right)$

$\left(\begin{array}{ccc|c} -1 & -6 & 12 & 6 \\ 0 & -8 & 12 & 6 \\ 0 & 12 & -18 & 0 \end{array} \right) \xrightarrow{\substack{F_2/2 \\ F_3/3}} \left(\begin{array}{ccc|c} -1 & -6 & 12 & 6 \\ 0 & -4 & 6 & 3 \\ 0 & 4 & -6 & 0 \end{array} \right) \xrightarrow{F_3+F_2} \left(\begin{array}{ccc|c} -1 & -6 & 12 & 6 \\ 0 & -4 & 6 & 3 \\ 0 & 0 & 0 & 3 \end{array} \right) E_3: 0=3$ S. Incompatible

Para $a=1$ $\left. \begin{array}{l} x - y + 2z = 1 \\ x - 2y = 0 \\ x + y - z = 1 \end{array} \right\} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & -2 & 0 & 0 \\ 1 & 1 & -1 & 1 \end{array} \right) \xrightarrow{\substack{F_2-F_1 \\ F_3-F_1}} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 2 & -3 & 0 \end{array} \right) \xrightarrow{F_3+2F_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & -7 & -2 \end{array} \right)$

$-7z = -2 \Rightarrow z = 2/7$
 $-y - 2z = -1 \Rightarrow y = 1 - 2z = 1 - 4/7 = 3/7$
 $x - y = 0 \Rightarrow x = y = 3/7$
 Sol $(3/7, 3/7, 2/7)$

Ej

15

$$\begin{cases} 3x - y + 2z = 1 \\ x + 4y + z = b \\ 2x - 5y + az = -2 \end{cases} \quad |A| = \begin{vmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ 2 & -5 & a \end{vmatrix} = 12a + 10 - 2 - 16 + 15 + a = 13a - 13 = 0$$

$a = 1$

(11)

Si $a \neq 1$ $\Rightarrow |A| \neq 0 \Rightarrow$ S. Compatible Determinado

Si $a \neq 1$

$$\begin{cases} 3x - y + 2z = 1 \\ x + 4y + z = b \\ 2x - 5y + z = -2 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 4 & 1 & b \\ 3 & -1 & 2 & 1 \\ 2 & -5 & 1 & -2 \end{array} \right) \begin{array}{l} F_2 - 3F_1 \\ F_3 - 2F_1 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 4 & 1 & b \\ 0 & -13 & -1 & 1-3b \\ 0 & -13 & -1 & -2-2b \end{array} \right) \begin{array}{l} \\ \\ F_3 - F_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 1 & b \\ 0 & -13 & -1 & 1-3b \\ 0 & 0 & 0 & -3+b \end{array} \right) \quad E_3: 0 = -3 + b$$

Si $a = 1$ y $b \neq 3$ $E_3: 0 = -3 + b \neq 0$ Contradicción. S. Incompatible

Si $a = 1$ y $b = 3$

$$\begin{cases} x + 4y + z = b \\ -13y - z = 1 - 3b \end{cases} \quad \begin{cases} x + 4y + z = 3 \\ -13y - z = -8 \end{cases} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} z = 8 - 13y$$

$$x + 4y + 8 - 13y = 3 \Rightarrow x = 3 - 4y - 8 + 13y \Rightarrow x = -5 + 9y$$

Sol $(-5 + 9y, y, 8 - 13y)$ S. Compatible indeterminado

Ej

11

$$\begin{cases} 6x + 4y + 2az = 2 \\ ax + y - z = 2 \\ 5x + 3y + 3z = 2a \end{cases} \quad |A| = \begin{vmatrix} 6 & 4 & 2a \\ a & 1 & -1 \\ 5 & 3 & 3 \end{vmatrix} = 18 + 6a^2 - 20 - 10a + 18 - 12a = 6a^2 - 22a + 16 = 0$$

$$3a^2 - 11a + 8 = 0 \quad a = \frac{11 \pm \sqrt{121 - 96}}{6} = \frac{11 \pm 5}{6} < \begin{array}{l} 16/6 = 8/3 \\ 6/6 = 1 \end{array}$$

Si $a \neq 8/3$ y $a \neq 1$ $\Rightarrow |A| \neq 0 \Rightarrow$ S. Compatible determinado

Si $a = 1$

$$\begin{cases} 6x + 4y + 2z = 2 \\ x + y - z = 2 \\ 5x + 3y + 3z = 2 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 6 & 4 & 2 & 2 \\ 5 & 3 & 3 & 2 \end{array} \right) \begin{array}{l} \\ F_2 - 6F_1 \\ F_3 - 5F_1 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -2 & 8 & -10 \\ 0 & -2 & 8 & 8 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -2 & 8 & -10 \\ 0 & 0 & 0 & 18 \end{array} \right)$$

$E_3: 0 = 18$ Contradicción. S. incompatible.

Si $a = 8/3$

$$\begin{cases} 6x + 4y + \frac{16}{3}z = 2 \\ \frac{8}{3}x + y - z = 2 \\ 5x + 3y + 3z = \frac{16}{3} \end{cases} \quad \begin{cases} 18x + 12y + 16z = 6 \\ 8x + 3y - 3z = 6 \\ 15x + 9y + 9z = 16 \end{cases} \quad \begin{cases} 9x + 6y + 8z = 3 \\ 8x + 3y - 3z = 6 \\ 15x + 9y + 9z = 16 \end{cases}$$

$$\left(\begin{array}{ccc|c} 9 & 6 & 8 & 3 \\ 8 & 3 & -3 & 6 \\ 15 & 9 & 9 & 16 \end{array} \right) \begin{array}{l} F_1 - F_2 \\ \\ F_3 - 15F_2 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 3 & 11 & -3 \\ 8 & 3 & -3 & 6 \\ 15 & 9 & 9 & 16 \end{array} \right) \begin{array}{l} \\ F_2 - 8F_1 \\ F_3 - 15F_1 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 3 & 11 & -3 \\ 0 & -21 & -91 & 30 \\ 0 & -36 & -156 & 61 \end{array} \right) \begin{array}{l} \\ \\ 36F_2 - 21F_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 3 & 11 & -3 \\ 0 & -21 & -91 & 30 \\ 0 & 0 & 0 & -201 \end{array} \right)$$

$E_3: 0 = -201$ Contradicción. S. Incompatible

(12)

Ej
12

$$\left. \begin{array}{l} x - y + z = a \\ 3x + 4y - 5z = 6 \\ x - y + bz = 2 \end{array} \right\} |A| = \begin{vmatrix} 1 & -1 & 1 \\ 3 & 4 & -5 \\ 1 & -1 & b \end{vmatrix} = 4b + 3 + 5 - 4 - 5 + 3b = 7b - 7 = 0$$

$$b = \frac{7}{7} = 1$$

Si $b \neq 1$ $\Rightarrow |A| \neq 0 \Rightarrow$ S. compatible determinado

$$\text{Si } b = 1 \quad \left. \begin{array}{l} x - y + z = a \\ 3x + 4y - 5z = 6 \\ x - y + z = 2 \end{array} \right\} \begin{array}{l} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & -1 & 1 & a \\ 3 & 4 & -5 & 6 \end{array} \right) \begin{array}{l} F_2 - F_1 \\ F_3 - 3F_1 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & a-2 \\ 0 & 7 & -8 & 0 \end{array} \right)$$

Si $b = 1$ y $a \neq 2$ $E_3: 0 = a - 2 \neq 0$ Contradicción. S. Incompatible

Si $b = 1$ y $a = 2$ $E_3: 0 = 0$

$$7y - 8z = 0 \Rightarrow y = \frac{8z}{7}$$

$$x - y + z = 2 \quad x = 2 + y - z = 2 + \frac{8z}{7} - z = \frac{14+z}{7}$$

Sol $(\frac{14+z}{7}, \frac{8z}{7}, z)$. S. compatible indeterminado.